12.4. MINOR ENERGY LOSSES

Whereas the major loss of energy or head is due to friction, the minor loss of energy (or head) includes the following cases:

1. **Loss of head due to sudden enlargement,**
2. **Loss of head due to sudden contraction,**
3. **Loss of head due to an obstruction in the pipe,**
4. **Loss of head at the entrance to a pipe,**
5. **Loss of head at the exit of a pipe,**
6. **Loss of head due to bend in the pipe, and**
7. **Loss of head in various pipe fittings.**

12·4·1 Loss of Head due to Sudden Enlargement

Fig. 12·4. shows a liquid flowing through a pipe which has *sudden enlargement*. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head).

Consider two sections 1 – 1(before enlargement) and 2 – 2 (after enlargement).

Let, A_1 = Area of pipe at section 1–1.

$= \frac{\pi}{4} D_1^2$ (where D_1 is the diameter of the pipe),

p_1 = Intensity of pressure at section 1–1,

V_1 = Velocity of flow at section 1–1,

$A_2 \left(= \frac{\pi}{4} D_2^2\right)$, p_2 and V_2 = Corresponding values at section 2–2,

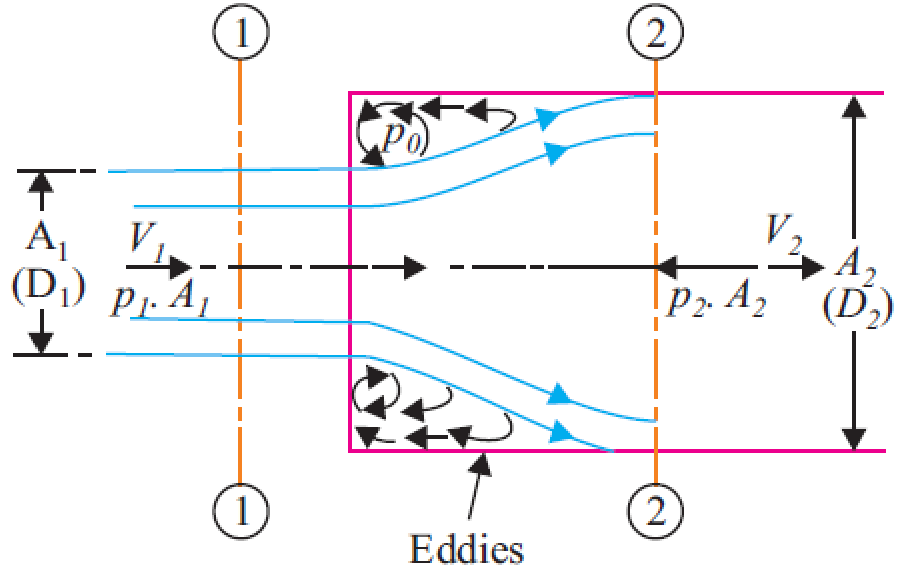

Fig. 12.4. Loss of head due to sudden enlargement.

p_0 = Intensity of pressure of the liquid eddies on the area $(A_2 - A_1)$, and

h_e = Loss of head due to sudden enlargement.

Applying Bernoulli's equation to sections 1–1 and 2–2, we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head due to sudden enlargement } (h_e)$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

Example 12.8. At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Calculate the rate of flow.

Solution. Diameter of the smaller pipe, $D_1 = 240 = \text{mm} = 0{\cdot}24$ m

Diameter of larger pipe, $D_2 = 480$ mm $= 0{\cdot}48$ m

Rise of hydraulic gradient, *i.e.*

$$\left(\frac{p_2}{w} + z_2\right) - \left(\frac{p_1}{w} + z_1\right) = 10 \text{ mm} = 0{\cdot}01 \text{ m}$$

$$\left[\text{The term } \left(\frac{p}{w} + z\right) \text{ prescribes the hydraulic gradient}\right]$$

Rate of flow, Q:

Applying Bernoulli's equation to small and large pipe sections (1–1 and 2–2), we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e \text{ (\textit{i.e.}, head lost due to sudden enlargement)} \quad ...(i)$$

But,

$$h_e = \frac{(V_1 - V_2)^2}{2g} \quad ...(ii)$$

From continuity equation, we have:

$$A_1 V_1 = A_2 V_2$$

$$\therefore \quad V_1 = \frac{A_2 V_2}{A_1} = \frac{\frac{\pi}{4} \times D_2^2}{\frac{\pi}{4} \times D_1^2} \times V_2 = \left(\frac{D_2}{D_1}\right)^2 \times V_2$$

or,

$$V_1 = \left(\frac{0.48}{0.24}\right)^2 \times V_2 = 4V_2$$

Substituting this value of V_1 in eqn. (*ii*), we get:

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now, substituting the values of he and V_1 in eqn. (*i*), we have:

$$\frac{p_1}{w} + \frac{(4V_2)^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

or,
$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{p_2}{w} + z_2\right) - \left(\frac{p_1}{w} + z_1\right)$$

or,
$$\frac{6V_2^2}{2g} = 0.01$$

or,
$$V_2 = \left(\frac{0.01 \times 2 \times 9.81}{6}\right)^{1/2} = 0.181 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2V_2 = \frac{\pi}{4} \times 0.48^2 \times 0.181 = \mathbf{0.03275\ m^3/s}$ **(Ans.)**

Example 12.9. A horizontal pipe 150 mm in diameter is joined by sudden enlargement to a 225 mm diameter pipe. Water is flowing through it at the rate of 0·05 m3/s. Find: **(i)** Loss of head due to abrupt expansion, **(ii)** Pressure difference in the two pipes, and **(iii)** Change in pressure if the change of section is gradual without any loss.

Solution. Diameter of the smaller pipe, $D_1 = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

Diameter of the larger pipe, $D_2 = 225 \text{ mm} = 0{\cdot}225 \text{ m}$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.225^2 = 0.03976 \text{ m}^2$$

$$\text{Discharge, } Q = 0{\cdot}05 \text{ m}^3/\text{s}$$

(*i*) Loss of head due to abrupt expansion, h_e:

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

where, V_1 and V_2 are the velocities of flow in the smaller and larger diameter pipes respectively.

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.01767} = 2.83 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.03976} = 1.26 \text{ m/s}$$

Hence,

$$h_e = \frac{(2.83 - 1.26)^2}{2 \times 9.81} = \mathbf{0.1256 \text{ m}} \text{ (Ans.)}$$

(ii) **Pressure difference in the two pipes :**

Applying Bernoulli's equation at the smaller and the larger pipe sections, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_e$$

or, $$\left(\frac{p_2 - p_1}{w}\right) = \frac{V_1^2 - V_2^2}{2g} - h_e$$ [$\because z_1 = z_2$, the pipe being horizontal]

or, $$\left(\frac{p_2 - p_1}{w}\right) = \frac{2.83^2 - 1.26^2}{2 \times 9.81} - 0.1256 = \mathbf{0.202\ m\ of\ water}$$ **(Ans.)**

The positive sign indicates that there is *gain* in pressure. Thus, although there is an energy loss, the pressure increases across a sudden flow of expansion.

(iii) **Change in pressure with gradual change of section :**

If the change of section is gradual *without loss,* then, gain in pressure,

$$\frac{p_2 - p_1}{w} = \frac{V_1^2 - V_2^2}{2g} = \frac{2.83^2 - 1.26^2}{2 \times 9.81} = \mathbf{0.327\ m\ of\ water}$$ **(Ans.)**

12·4·2. Loss of Head due to Sudden Contraction

Due to sudden contraction, the stream lines converge to a minimum cross-section called the *vena-contract* and then expand to fill the downstream pipe (Fig. 12·5.)

Let, A_c = Area of flow at section C–C,

V_c = Velocity of flow at section C–C,

A_2 = Area of flow at section 2–2,

V_2 = Velocity of flow at section 2–2, and

h_c = Loss of head due to sudden contraction.

Loss of head due to sudden contraction = Loss up to vena-contracta + loss due to sudden enlargement beyond vena-contracta

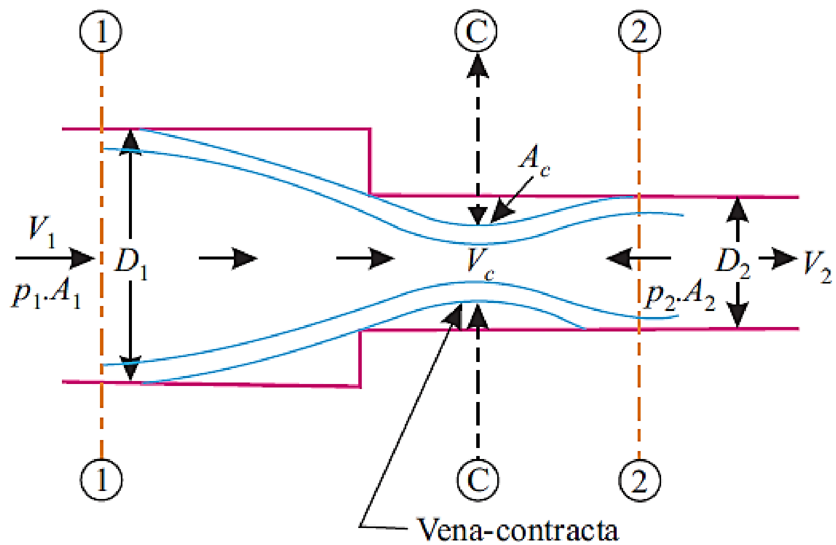

Fig. 12.5

or, $$h_c = \text{Negligibly small} + \frac{(V_c - V_2)^2}{2g} \qquad \ldots(i)$$

$$\boxed{h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2}$$

Example 12.13. A horizontal pipe carries water at the rate of 0·04 m³/s. Its diameter, which is 300 mm reduces abruptly to 150 mm. Calculate the pressure loss across the contraction. Take the co-efficient of contraction = 0·62.

Solution. Diameter of the large pipe, $D_1 = 300 \text{ mm} = 0{\cdot}3 \text{ m}$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

Diameter of the small pipe, $D_2 = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.15^2 = 0.1767 \text{ m}^2$$

Discharge, $Q = 0{\cdot}04 \text{ m}^3/\text{s}.$

Co-efficient of contraction, $C_c = 0{\cdot}62$

Pressure loss across the contraction, $(p_1 - p_2)$:

From continuity equation, we have:

$$A_1V_1 = A_2V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.04}{0.0707} = 0.566 \text{ m/s}$$

and,

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.26 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c \qquad \text{...(i)}$$

But, $z_1 = z_2$ ...because the pipe is horizontal and head loss due to contraction (h_c) is given as :

$$h_c = \left[\frac{1}{C_c} - 1\right]^2 \frac{V_2^2}{2g} = \left[\frac{1}{0.62} - 1\right]^2 \times \frac{2.26^2}{2 \times 9.81} = 0.0978$$

Substituting these values in eqn. (i), we get:

$$\frac{p_1}{w} + \frac{0.566^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{2.26^2}{2 \times 9.81} + 0.0978$$

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} = \frac{2.26^2}{2 \times 9.81} + 0.0978 - \frac{0.566^2}{2 \times 9.81}$$

$$= 0{\cdot}26 + 0{\cdot}0978 - 0{\cdot}016 = 0{\cdot}3418$$

Hence, $$p_1 - p_2 = w \times 0{\cdot}3418 = 9{\cdot}81 \times 0{\cdot}3418$$

$$= \mathbf{3{\cdot}35\ kN/m^2} \quad \textbf{(Ans.)}$$

Example 12.15. When a sudden contraction is introduced in a horizontal pipeline from 500 mm diameter to 250 mm diameter, the pressure changes from 105 kN/m² to 69 kN/m². If the co-efficient of contraction is assumed to be 0·65, calculate the water flow rate. Following this if there is sudden enlargement from 250 mm to 500 mm and if the pressure at the 250 mm section is 69 kN/m², what is the pressure at the 500 mm enlarged portion ?

Solution. Diameter of the large pipe, $D_1 = 500 \text{ m} = 0{\cdot}5 \text{ m}$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} \times 0.5^2 = 0.1963 \text{ m}^3$$

Diameter fo the small pipe, $D_2 = 250 \text{ mm} = 0{\cdot}025 \text{ m}$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.25^2 = 0.04908 \text{ m}^2$$

Pressure inside the large pipe, $p_1 = 105 \text{ kN/m}^2$

Pressure inside the small pipe, $p_2 = 69 \text{ kN/m}^2$

Co-efficient of contraction, $C_c = 0{\cdot}65$

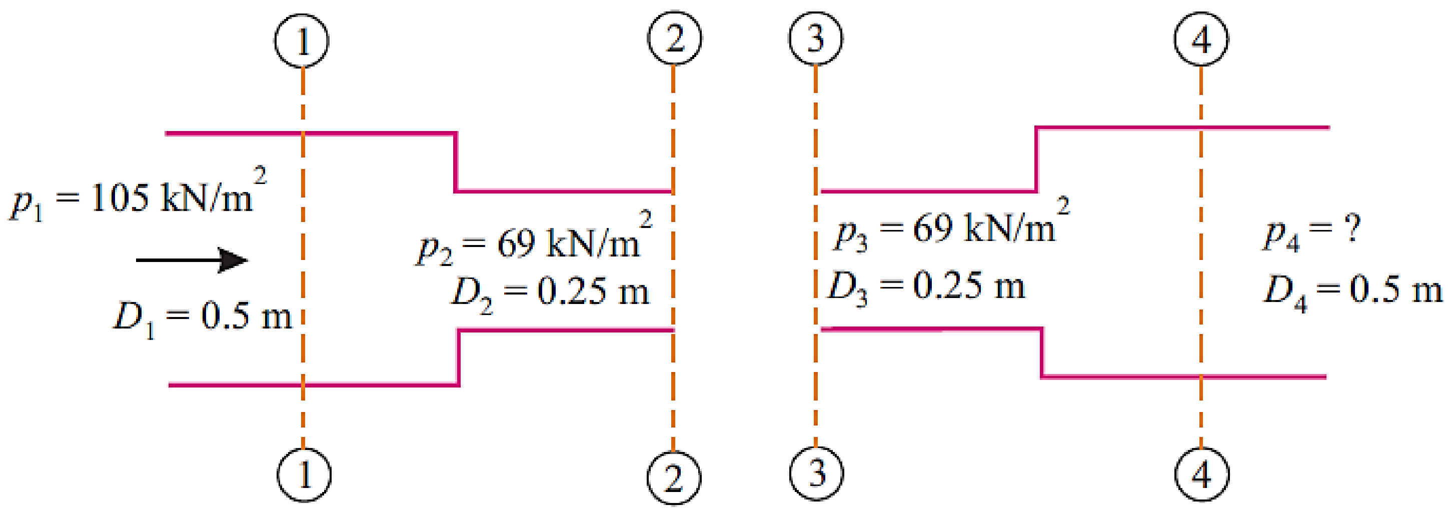

Fig. 12.6

(*i*) Flow rate, Q :

Head lost due to contraction is given by:

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1.0\right)^2 = \frac{V_2^2}{2g}\left[\frac{1}{0.65} - 1.0\right]^2 \qquad \text{[Eqn. (12·3)]}$$

$$= 0.2899\,\frac{V_2^2}{2g} \qquad \text{...}(i)$$

From continuity considerations, we have:

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1} \times V_2 = \frac{(\pi/4) \times D_2^2}{(\pi/4) \times D_1^2} \times V_2$$

or,

$$V_1 = \left(\frac{0.25}{0.50}\right)^2 \times V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation at 1-1 and 2-2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_c$$

But,

$$z_1 = z_2 \qquad \text{...the pipe being horizontal.}$$

$\therefore$

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + h_c$$

$$\therefore \qquad \frac{p_1}{w}+\frac{V_1^2}{2g}=\frac{p_2}{w}+\frac{V_2^2}{2g}+h_c$$

Substituting the values, we get:

$$\frac{105}{9.81}+\frac{(V_2/4)^2}{2\times 9.81}=\frac{69}{9.81}+\frac{V_2^2}{2\times 9.81}+0.2899\,\frac{V_2^2}{2\times 9.81}$$

or, $$210+\frac{V_2^2}{16}=138+V_2^2+0.2899\,V_2^2$$

or, $$72=1.2899\,V_2^2-\frac{V_2^2}{16}=1.2274\,V_2^2$$

$$V_2 = 7{\cdot}66 \text{ m/s}$$

Hence, rate of flow, $Q = A_2V_2 = 0{\cdot}04908 \times 7{\cdot}66 = \mathbf{0{\cdot}376\ m^3/s}$ **(Ans.)**

(ii) **Pressure at the enlarged section, p_4 :**

Applying Bernoulli's equation at the sections 3-3 and 4-4, we get:

$$\frac{p_3}{w}+\frac{V_3^2}{2g}+z_3=\frac{p_4}{w}+\frac{V_4^2}{2g}+z_4+h_e \text{ (loss of head due to sudden enlargement)}$$

But, $$p_3 = 69 \text{ kN/m}^2$$

$$V_3 = V_2 = 7{\cdot}66 \text{ m/s}$$

$$V_4 = V_1 = \frac{V_2}{4}=\frac{7.66}{4}=1.915 \text{ m/s}$$

$$z_3 = z_4$$

And, $$h_e=\frac{(V_3-V_4)^2}{2g}=\frac{(7.66-1.915)^2}{2\times 9.81}\text{ m}=1.68\text{ m}$$

Substituting the values in the above equation, we get:

$$\frac{69}{9.81}+\frac{7.66^2}{2\times 9.81}=\frac{p_4}{9.81}+\frac{(1.915)^2}{2\times 9.81}+1.68$$

$$7.033+2.99=\frac{p_4}{9.81}+0.187+1.68$$

or, $$\mathbf{p_4 = 80\ kN/m^2}$$ **(Ans.)**

12·4·3 Loss of Head due to Obstruction in Pipe

Refer to Fig. 12·7. The loss of energy due to an obstruction in pipe takes place on account of the reduction in the cross-sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream beyond the obstruction. Head loss due to obstruction (*h*obs.) is given by the relation:

$$h_{obc.} = \left[\frac{A}{C_c(A-a)}\right]^2 \frac{V^2}{2G}$$

where, A = Area of the pipe, a = Maximum area of obstruction, and

V = Velocity of liquid in pipe.

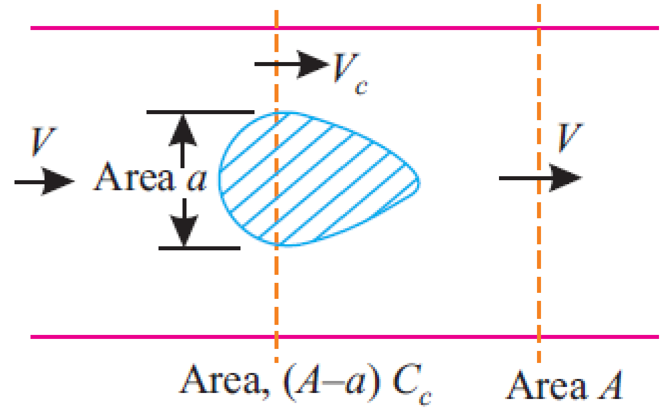

Fig. 12.7

12·4·4 Loss of Head at the Entrance to Pipe

Loss of head at the entrance to pipe (h_i) is given by the

relation : $$h_i = 0.5\,\frac{V^2}{2G}$$

where, V = Velocity of liquid in pipe.

12·4·5 Loss of Dead at the Exit of a Pipe

Loss of head at the exit of a pipe is denoted by *h0* and is given by the relation:

$$h_o = \frac{V^2}{2G}$$

where, V = Velocity at outlet of pipe.

12·4·6 Loss of Head due to Bend in the Pipe

In general the loss of head in bends (*hb*) provided in pipes may be expressed as:

$$h_o = k\,\frac{V^2}{2G}$$

where, V = Mean velocity of flow of fluid, and
and, k = Co-efficient of bend; it depends upon *angle of bend, radius of curvature of bend* and *diameter of pipe.*

12·4·7 Loss of Head in Various Pipe Fittings

The loss of head in the various pipe fittings (such as valves, couplings, etc.) may also be represented as :

$$h_{fitting} = k\,\frac{V^2}{2G}$$

where, V = Mean velocity flow in the pipe, and k = value of the co-efficient; it depends on the type of the pipe fitting.

12.6. PIPES IN SERIES OR COMPOUND PIPES

Fig. 12·15 shows a system of pipes in series.

As the rate of flow (Q) of water through each pipe is same, therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, The difference in liquid surface levels = Sum of the various head losses in the pipes

i.e.,
$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^3}{2g} \quad ...(i)$$

where,

h_i = Head loss at entrance $= \dfrac{0.5V_1^2}{2g}$

h_{f_1} = Head loss due to friction in pipe 1 $= \dfrac{4f_1L_1V_1^2}{D_1 \times 2g}$

h_c = Head loss at contraction $= \dfrac{0.5V_2^2}{2g}$

h_{f_2} = Head loss due to friction in pipe 2 $= \dfrac{4f_2L_2V_2^2}{D_2 \times 2g}$

h_e = Head loss due to enlargement $= \dfrac{(V_2 - V_3)^2}{2g}$

h_{f_3} = Head loss due to friction in pipe 3 $= \dfrac{4f_3L_3V_3^2}{D_3 \times 2g}$

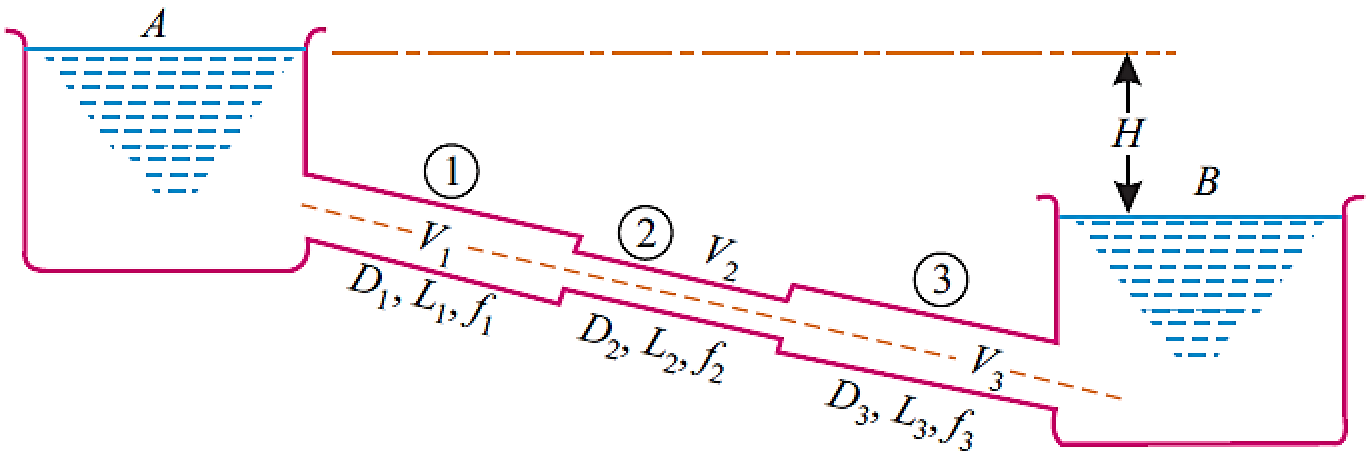

Fig. 12.15. Pipes in series.

Substituting the values in (*i*), we have:

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g} \quad ...(12{\cdot}9)$$

If *minor losses are neglected*, then above equation becomes:

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad ...(12{\cdot}10)$$

If, $f_1 = f_2 = f_3 = f$, then:

Example 12.22. Three pipes of diameters 300 mm, 200 mm and 400 mm and lengths 450 m, 255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if co-efficients of friction are 0·0075, 0·0078 and 0·0072 respectively considering:

(i) Minor losses also, and **(ii)** Neglecting minor losses.

Solution. Pipe 1 : $L_1 = 450$ m, $= D_1 = 300$ mm $= 0{\cdot}3$ m, $f_1 = 0{\cdot}0075$

Pipe 2 : $L_2 = 255$ m, $D_2 = 200$ mm $= 0{\cdot}2$ m, $f_2 = 0{\cdot}0078$

Pipe 3 : $L_3 = 315$ m, $D_3 = 400$ mm $= 0{\cdot}4$ m, $f_3 = 0{\cdot}0072$

Difference of water level, $H = 18$ m.

***(i)* Considering minor losses :**

Let V_1, V_2 and V_3 be the velocities in Ist, 2nd, and 3rd pipe respectively.

From continuity considerations, we have:

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(\pi/4)\times D_1^2}{(\pi/4)\times D_2^2}\times V_1 = \frac{D_1^2}{D_2^2}\times V_1 = \left(\frac{0.3}{0.2}\right)^2 V_1 = 2.25\,V_1$$

$$\text{and,} \quad V_3 = \frac{A_1V_1}{A_3} = \frac{(\pi/4)\times D_1^2}{(\pi/4)\times D_3^2}\times V_1 = \frac{D_1^2}{D_3^2}\times V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\,V_1$$

We know that:

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1\times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2\times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3\times 2g} + \frac{V_3^2}{2g} \qquad \text{...[Eqn. (12·9)]}$$

$$18 = \frac{0{\cdot}5V_1^2}{2g} + \frac{4 \times 0.0075 \times 450 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25\ V_1)^2}{2g} + \frac{4 \times 0.0078 \times 255 \times (2.25V_1)^2}{0.2 \times 2g}$$

$$+ \frac{(2.25\ V_1 - 0.5625\ V_1)^2}{2g} + \frac{4 \times 0.0072 \times 315 \times (0.5625\ V_1)^2}{0.4 \times 2g} + \frac{(0.5625\ V_1)^2}{2g}$$

$$18 = \frac{V_1^2}{2g}(0.5 + 45 + 2.53 + 201.4 + 2.847 + 7.176 + 0.316)$$

$$= 259.77\ \frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{259.77}} = 1.166 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_1 \times V_1 = (\pi/4) \times 0{\cdot}3^2 \times 1{\cdot}166 =$ **$0{\cdot}0824$ m^3/s** **(Ans.)**

(*ii*) Neglecting minor losses :

We know that, $$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad \text{...[Eqn. (12·10)]}$$

$$18 = \frac{V_1^2}{2g}\left(\frac{4 \times 0.0075 \times 450}{0{\cdot}3} + \frac{4 \times 0.0078 \times 255 \times 2.25^2}{0{\cdot}2} + \frac{4 \times 0.0072 \times 315 \times (0.5625)^2}{0{\cdot}4}\right)$$

$$= \frac{V_1^2}{2g}(45 + 201.4 + 7.176) = 253.57 \times \frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{253.57}} = 1.18 \text{ m}$$

$\therefore$ Discharge, $Q = A_1V_1 = (\pi/4) \times 0{\cdot}3^2 \times 1{\cdot}18 =$ **$0{\cdot}0834$ m^3/s** **(Ans.)**